Detecting defects in composite beams and plates using Bayesian inference

Defect Detection as an Inverse Problem

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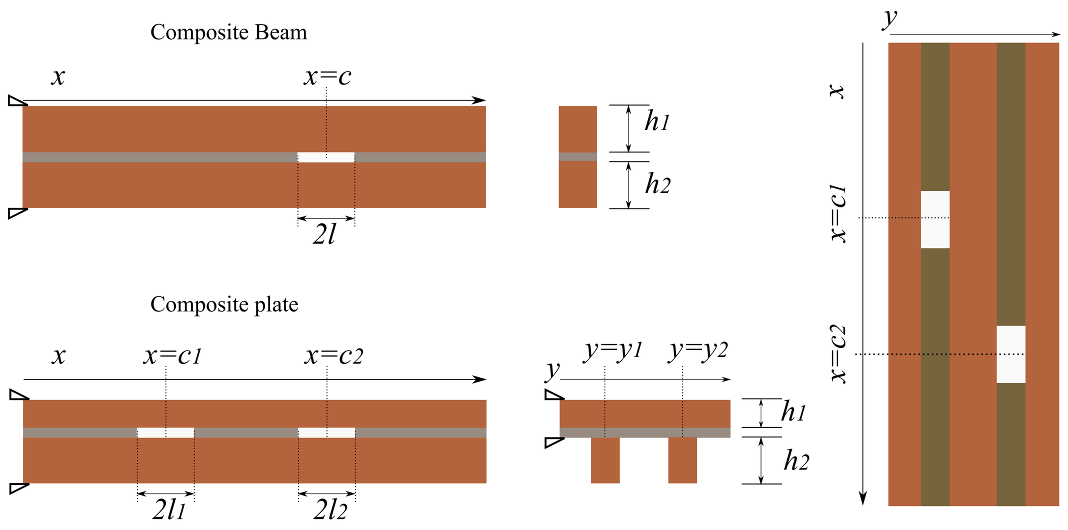
Abstract

The topic of this paper is an inverse problem of identifying defects in composite beams and plates. Assuming Gaussian errors in measurements, the Bayesian inference is performed for unknown parameters, and the most probable physical representations of detects are estimated. We use the natural frequencies of the beam/plate to estimate the position and the size of the defects. We propose that the bonding within the beam and plate can be modelled as added rigidity, which can be incorporated as an extra energy to the conventional strain energy. Standard Monte-Carlo simulation will then give the probabilistic properties of the natural frequencies of the beam/plate. The more prior information about the defects is limited, and thus we estimate the posterior distribution using the trans-dimensional Bayesian method, which lets us make an inference of different types of defects.

Introduction



The bending motion of this Euler beams will be represented using the eigenfunctions of the motion, which satisfy the boundary conditions. The eigenfunctions of the clamped beam are

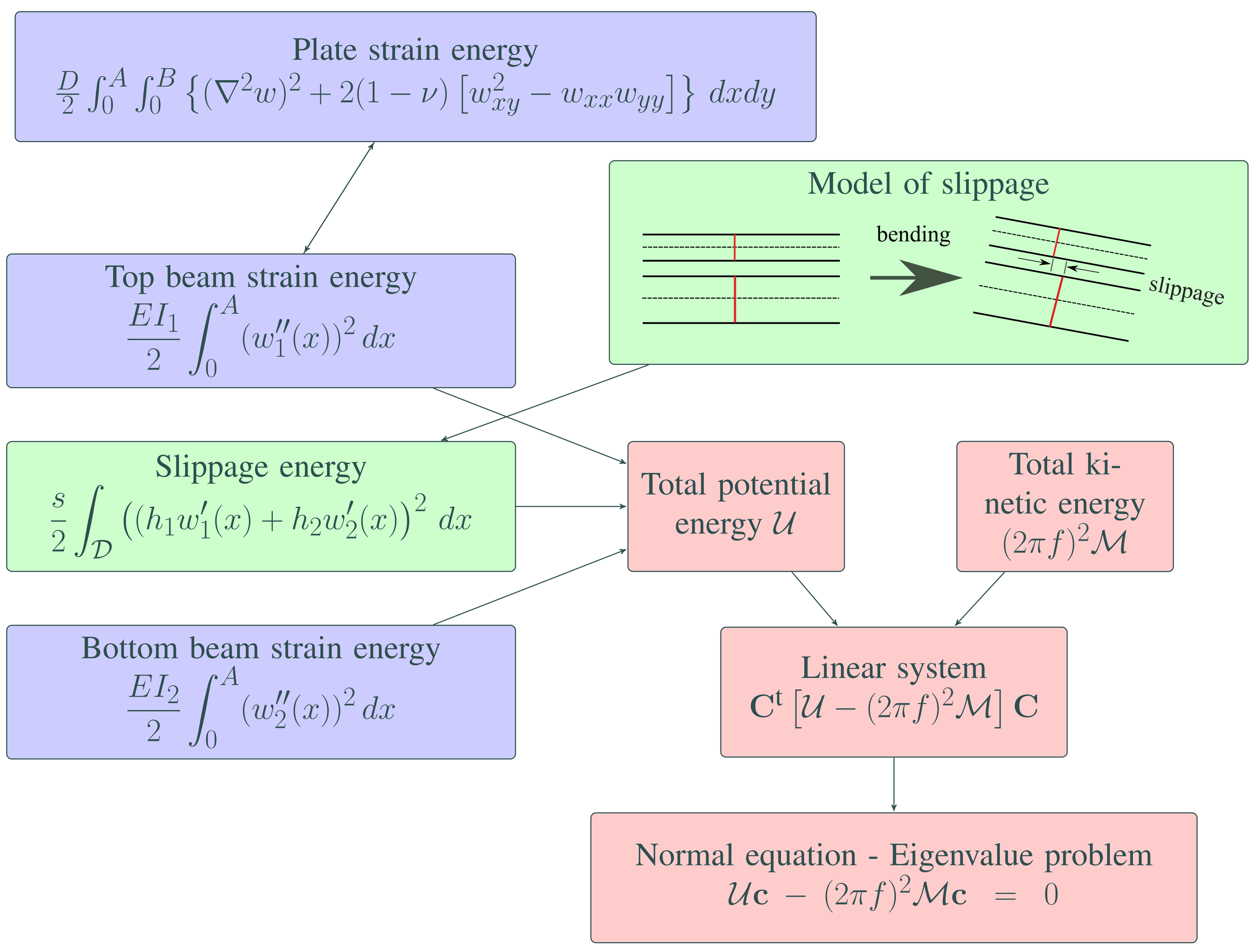
$$\psi_m(x) = \sqrt{\frac{1}{A}}\left[\cosh k_m x - \cos k_m x - \gamma_m \left(\sinh k_m x - \sin k_m x\right)\right], \quad m = 1, 2, ...$$

where

$$\gamma_m = \frac{\cos k_m A + \cosh k_m A}{\sin k_m A - \sinh k_m A}, \quad \cos k_m A \cosh k_m A = -1$$

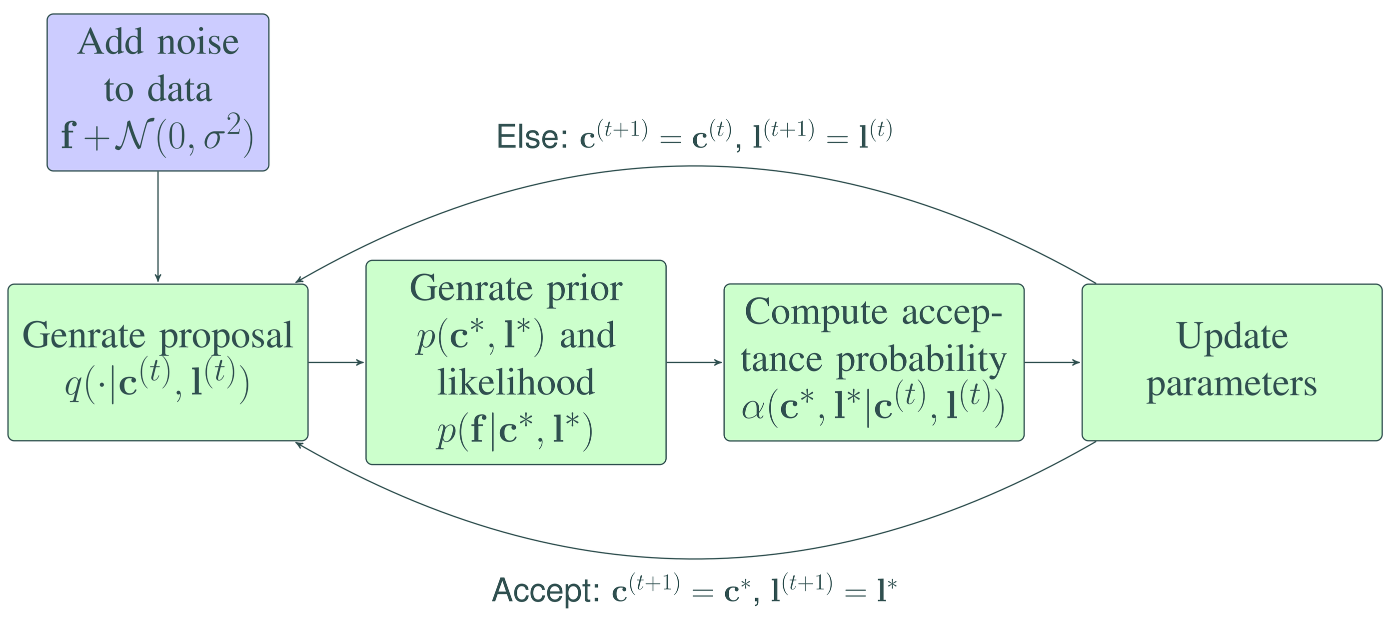
The vertical displacement of the beam is $w(x) = \sum_{m=1}^{N} C_m \psi_m(x)$ and of the plate is $w(x,y) = \sum_{m,n=1}^{N} C_{mn}\psi_m(x)\phi_n(y)$. A linear system can be formed for the vector $\mathbf{C} = \{C_1, C_2, ..., C_N\}$ for a beam and $\mathbf{C} = \{C_{11}, C_{12}, ..., C_{NN}\}$ for a plate.

Forward problem: Eigenvalue Problem



MCMC Simulations

- Position of de-lamination $\mathbf{c} = \{c_1, c_2, ...c_{N_d}\}$, Length of de-lamination $\mathbf{l} = \{l_1, l_2, ...l_{N_d}\}$
- Natural frequencies $\mathbf{f} = \{f_1, f_2, ...f_{10}\}$



Inverse Problem: MCMC - Metropolis-Hastings

We use the Bayesian inference to estimate the distribution for unknown defects, $(\mathbf{c}, \mathbf{l})$, from an observed set of natural frequencies, $\mathbf{f} = \{f_i\}_{i=1}^{N_f}$. This is called the posterior distribution and denoted by $p(\mathbf{c}, \mathbf{l}|\mathbf{f})$. If a probability for $\mathbf{f}$ is non-zero, the posterior is represented by the Bayes' rule, which is

$$p(\mathbf{c}, \mathbf{l}|\mathbf{f}) = \frac{p(\mathbf{f}|\mathbf{c}, \mathbf{l})\, p(\mathbf{c}, \mathbf{l})}{p(\mathbf{f})} \propto p(\mathbf{f}|\mathbf{c}, \mathbf{l})\, p(\mathbf{c}, \mathbf{l})$$

where $p(\mathbf{f}|\mathbf{c}, \mathbf{l})$ is the likelihood and $p(\mathbf{c}, \mathbf{l})$ is a prior. The posterior distribution is proportional to the likelihood and prior.

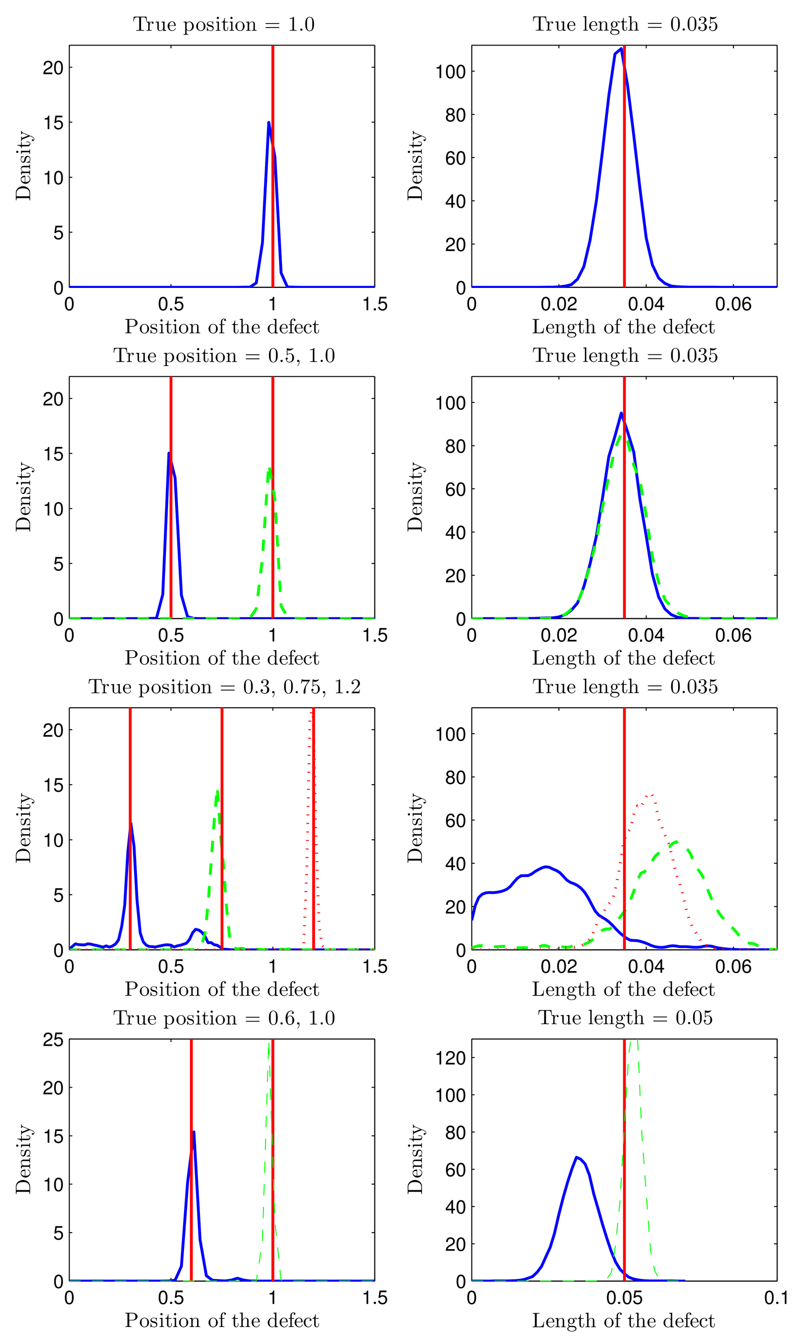
At the t-th iteration, $(\mathbf{c}^{(t)}, \mathbf{l}^{(t)})$ is updated as the followings.

1. Generate the proposals, $(\mathbf{c}^*, \mathbf{l}^*) \sim q(\cdot|\mathbf{c}^{(t)}, \mathbf{l}^{(t)})$.
2. Compute the prior $p(\mathbf{c}^*, \mathbf{l}^*)$ and likelihood $p(\mathbf{f}|\mathbf{c}^*, \mathbf{l}^*)$ in for $\mathbf{c}^*$ and $\mathbf{l}^*$.
3. Compute the acceptance probability, $\alpha(\mathbf{c}^*, \mathbf{l}^*|\mathbf{c}^{(t)}, \mathbf{l}^{(t)})$

$$\alpha(\mathbf{c}^*, \mathbf{l}^*|\mathbf{c}^{(t)}, \mathbf{l}^{(t)}) = \min\left(1, \frac{p(\mathbf{f}|\mathbf{c}^*, \mathbf{l}^*)p(\mathbf{c}^*, \mathbf{l}^*)q(\mathbf{c}^{(t)}|\mathbf{c}^*)q(\mathbf{l}^{(t)}|\mathbf{l}^*)}{p(\mathbf{f}|\mathbf{c}^{(t)}, \mathbf{l}^{(t)})p(\mathbf{c}^{(t)}, \mathbf{l}^{(t)})q(\mathbf{c}^*|\mathbf{c}^{(t)})q(\mathbf{l}^*|\mathbf{l}^{(t)})}\right)$$

4. Accept $\mathbf{c}^*$ and $\mathbf{l}^*$ with $\alpha(\mathbf{c}^*, \mathbf{l}^*|\mathbf{c}^{(t)}, \mathbf{l}^{(t)})$. If they are accepted, set $\mathbf{c}^{(t+1)} = \mathbf{c}^*$ and $\mathbf{l}^{(t+1)} = \mathbf{l}^*$. Otherwise $\mathbf{c}^{(t+1)} = \mathbf{c}^{(t)}$ and $\mathbf{l}^{(t+1)} = \mathbf{l}^{(t)}$.

Results



	notation	value
length	A	1.5 m
width	B	0.3 m
mass density	m	500 kgm^{-3}
Young's modulus	E	14 GPa
thickness	h_1	0.01 m
thickness	h_2	0.1 m
beam width	h_3	0.05 m
Poisson ratio	ν	0.4
slippage constant	s	10^8 Nm^{-1}
location of the beams	y_1, y_2	0.15 m, 0.25 m

Table 1: Notations and values of the physical parameters

Model	Parameter	Mean	95% confidence interval	True value
One defect	Position (c)	0.9882	[0.9382 , 1.0324]	1
	Length (l)	0.0337	[0.0266 , 0.0407]	0.035
Two defects	Position (c)	0.5023	[0.4625 , 0.5495]	0.5
		0.9856	[0.9301 , 1.0327]	1
	Length (l)	0.0341	[0.0253 , 0.0421]	0.035
		0.0346	[0.0254 , 0.0437]	0.035
Three defects	Position (c)	0.3582	[0.0628 , 0.6902]	0.3
		0.7294	[0.6691 , 0.7917]	0.75
		1.1925	[1.1637 , 1.2257]	1.2
	Length (l)	0.0171	[0.0008 , 0.0402]	0.035
		0.0444	[0.0175 , 0.0603]	0.035
		0.0399	[0.0288 , 0.0503]	0.035
Plate	Position (c)	0.6085	[0.5564 , 0.6723]	0.6
		0.9823	[0.9494 , 1.0161]	1
	Length (l)	0.0354	[0.0235 , 0.0475]	0.05
		0.0528	[0.0473 , 0.0582]	0.05

Table 2: Parameter estimate result using the MCMC method.

Conclusions

1. The natural frequencies are computed efficiently using the eigenfunction expansion of the bending vibration of the structures.
2. The accuracy of the de-lamination near the clamped edge is lower than others. The de-laminations in composite plate are more difficult to estimate, though the positions are estimated accurately.
3. The estimates of the length of the de-lamination show certain bias either to larger or smaller values. The reason is yet unknown to the authors.
4. It remains to be seen how the results in this paper can be scaled to other materials and sizes and without the prior knowledge of the number of the de-laminations.