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Lack of diffuseness in wave propagation in lightweight joist-floors

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ABSTRACT

It is common to assume the wave field to be diffuse in often used simplified calculation procedures in building acoustics, such as Statistical Energy Analysis (SEA) or energy flow methods. It is a well known that such diffuseness assumption is not always valid. This paper presents one such example. The structures under consideration are rib-reinforced plate structures. These may be a lightweight floor/joist structure in a building or hull of a ship. The diffuseness or rather the lack of diffuseness, in such 'ribbed-plate' structures is examined. The amount of power transported at an angle is calculated by means of the structural intensity in that direction

1 INTRODUCTION

The recent drive to develop of new building systems is intensive. These systems are often lightweight constructions, and developed to be used in load-bearing structures and dwellings. One of the main drawbacks with these lightweight building systems is the poor sound insulation. Without proper sound insulation, the tenant will not accept the building system since the system does not meet the demands of the society (represented in the building codes). Therefore the building system will not be used. The term 'lightweight building systems' often means timber frame structures, but also includes lightweight steel frames structures. The use of wooden frames in buildings is often recognized as an environmentally and economically sensible building technique.

It is then important to have theoretical models of the acoustic and vibration behaviour of the building structure. These models can be simplified or detailed. The model used in this paper is detailed in its nature. Therefore the model gives information and lets us confirm whether the assumptions about the simplified models are valid or not. In this paper, we use the typical *ribbed* structure figure (1) as an example. The deflection of the structure due to an external excitation force will be computed from the prescribed material property of the individual components, the joists and the plate, rather than using implicit parameters such as structural stiffness for the whole structure as opposed to the separate stiffness for each component.

The simplified models are on the other hand often based on energy flow considerations or, more elaborate, SEA, for example by Lyon and DeJong [5] or Craik [2]. Assuming a diffuse acoustic field is praxis in SEA and energy flow based calculations used in acoustics. It

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may be the case both in case of an airborne field, as in classical Sabine theory for room acoustics, or the vibration field of a structure using SEA.

The authors feel that there exists some confusion over the definition of 'diffuseness'. We here attempt to give a clear definition, which is used in this paper. In a diffuse field *each angle of propagation of a plane wave has the same probability, all phase angles have the same probability, and the intensity is uniformly distributed over all angles*. There is also a point stating that the energy is evenly spread in the structure/medium considered, but in my opinion that is a separate statement (about homogeneity). In Dictionary of acoustics [6], it is stated that diffuse field[3] is an idealized sound field that consists of infinitely many uncorrelated plane progressive waves, with their intensity uniformly distributed with respect to direction. The resultant acoustic intensity is therefore zero.

At a point in a diffuse sound field the quantity $cw/4$, where w represents the mean acoustic energy density and c is the speed of sound. Physically, $cw/4$ is the mean rate per unit surface area, at which energy in a diffuse field is incident on an imaginary plane surface from one side only. Hence, by assuming a diffuse field it is possible to determine the intensity hitting a surface (or line) in relation to the acoustic energy. The diffuse field is thus a basic assumption required in SEA or energy flow based calculations.

Several prediction models for flanking transmission in homogeneous structures have been available for a long time. These existing models mainly use energy flow methods and SEA. In the 90's, the models by Gerretsen [3,4], were put together in the CEN (European Committee for Standardization (Comite Europeen de Normalisation)) building acoustics prediction model EN 12354 [1]. In EN 12354, measurements of acoustic parameters such as sound insulation and structural reverberation time of the building elements are used as input data to the model. (The relationship between SEA and EN 12354 is discussed in [7].) EN 12354 is thus developed for homogeneous structures. However, there is an ongoing work (in an ad hoc group) to develop EN 12354 to be suitable also for the lightweight structures. The problems with such development can be described by the following four categories. We consider the third item in the list.

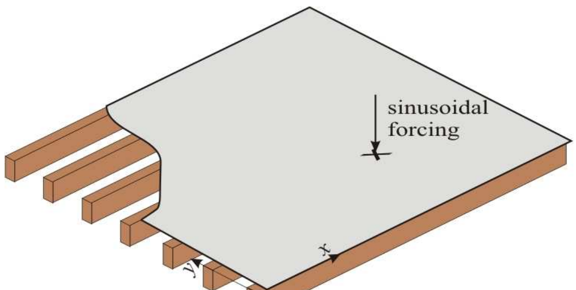

Figure 1: Drawing depicting the design of a joist floor. The structure has equally spaced joists running parallel to each other. The origin of the coordinate system is placed at the corner of the structure.

1. Heavy vs. lightweight: the radiation is not taken into account correctly for lightweight structures.
2. Low attenuation vs. high attenuation: the vibration field will be somewhat non-diffuse for high attenuation. The problem here is that the squared-average vibration velocity $<v^2>$ has a more complicated relationship to the structural TL than what is assumed so far in EN 12354. We propose that 'attenuation' is a more

[3] This definition implies that the single-point and two-point statistics of the pressure field are independent of both absolute position and orientation. Hence the field is statistically homogeneous and isotropic.

suitable word than damping here because 'attenuation' includes the periodic/nearly periodic effects.

3. Isotropic vs. orthotropic: the attenuation and/or bending stiffness is directional for orthotropic cases, which is caused by the periodicity in the structure. The vibration field will be strongly non-diffuse.
4. Single layer vs. double layer (multi layer): the problem here is twofold, first, which R-value is to be used, and second, how transmission coefficient is determined.

2 COMPUTATION OF DEFLECTION

2.1 Derivation of Lagrangian of the structure

We consider the vibration of the joist-floor depicted in figure (1). The amplitude of the deflection of the plate and the joist beams may be assumed to be small. Furthermore the dimension of the plate and the beams lets us model them as a thin (Kirchhof) plate and Euler beams. Therefore the vibration of the components is expressed only by the vertical deflection, which are denoted by $w_0(x,y)$ and $w_1(x,j)$, $j=1,2,...,S_1$ for the plate and the j'th beam, respectively. Note that the structure spans in the (x,y)-plane and the joists are running in the x-direction.

We compute the functions w_0 and w_1 using the variational formulation. The total energy in the structure will be formulated as a functional of w_0 and w_1, then the first variance will be taken. The total energy in the structure is the sum of strain, kinetic energy and the work due to the excitation force. We have the energy in the time period $[0,T]$,

$$S(w)=\int_0^T\left[K(t)+W(t)-P(t)\right]dt$$

where $\boldsymbol{K}$ and $\boldsymbol{P}$ are the kinetic and strain energy, and $\boldsymbol{W}$ is the work done on the structure.

The strain energy in the structure is the sum of the strain energy of the plate and the joists. The strain energy in the plate is

$$P_0=\frac{D}{2}\int_0^A\int_0^B\left\{(\Delta w_0)^2+2(1-\nu)\left[\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}-\left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2\right]\right\}dxdy$$

where D and ν are flexural rigidity and Poisson ratio of the plate, respectively. For the joists we have

$$P=\sum_{j=1}^{S_1}\frac{EI}{2}\int_0^A\left(\frac{\partial^2 w_1}{\partial x^2}\right)^2 dx$$

where E and I are Young's modulus and the moment of inertia of the joists, which are assumed to be same for all the joist. The dimension of the structure is denote by A and B for the width and the length, respectively.

The kinetic energy for the plate and the beams is

$$K=\frac{m_0}{2}\int_0^A\int_0^B\dot{w}_0^2\,dxdy+\frac{m_1}{2}\sum_j\int_0^A\dot{w}_1^2\,dx$$

where m_0 and m_1 are the mass density of the plate per unit area and of the beam per unit length, respectively. The dot indicates the time derivative.

The work done to the structure is determined by F, which denotes the excitation force distribution on the plate. We here excite the plate at a point (x_0,y_0) with 1 Newton and radial frequency ω, then we have

$$F(x,y,t)=\delta(x-x_0,y-y_0)\cos\omega t\,.$$

We are considering a linear system, hence the vibration the structure can be decomposed into a frequency part and a mode shape of $w_0(x,y)$ and $w_1(x,j)$, i.e.,

$$w_0(x,y,t) = w_0(x,y)\cos\omega t$$
$$w_1(x,j,t) = w_1(x,j)\cos\omega t,\quad j = 1,2,\ldots,S_1$$

Note that the same notation w_0 and w_1 are used for the time-dependent and the mode shape functions because there is no risk of confusion.

2.2 Method of solution

We consider the vibration of the structure in the non-dimensional space and time. In order to find the solution to the variational form given in the previous subsection, we define the boundary conditions for the structure and the modal expansion of the deflection functions.

We assume that the structure is simply supported, that is, the edge of the plate and the ends of the joists are pinned. Therefore the displacement and the bending moment are zero at the boundary of the structure.

We assume that the joists are also simply supported. Then, w_1 satisfies the same conditions for the displacement and the derivatives at $x=0,A$. Because of the rectangular shape of the structure and the above conditions, the deflection w_0 (and w_1) can be expressed using the Fourier series, more specifically Fourier sine-series. Therefore we have

$$w_0(x,y) = \sum_{m,n=1}^{\infty} c^0_{mn}\phi_m(x)\psi_n(y)$$

and the deflection of the j'th joist is

$$w_1(x,j) = \sum_{m=1}^{\infty} c^1_{mj}\phi_m(x)$$

where the modes are

$$\phi_m(x) = \sqrt{\frac{2}{A}}\sin k_m x,\quad \psi_n(x) = \sqrt{\frac{2}{B}}\sin \kappa_n x,$$

and the spatial wave numbers are $k_m=\pi m/A$ and $\kappa_n=\pi n/B$. The number of modes will be truncated to be N in order to formulate finite system, which can be solved numerically.

The coefficient vectors $\mathbf{c}_0$ and $\mathbf{c}_1$ are related by where the beams are attached to the plate, which leads to

$$\mathrm{c}_1 = \mathrm{J}\mathrm{c}_0$$

where matrix $\mathbf{J}$ represents the following relationship

$$w_1(x,j) = w_0(x,y_j) \Rightarrow c^1_{mj} = \sum_{n=1}^{N} c^0_{mn}\psi_n(y_j)$$

Note that the orthogonality of ϕ_m is used to obtain the second equation above.

The coefficients c^0_{mn} can be found by substituting the above equations and taking the partial derivatives of the Lagrangian with respect to c^0_{mn}. For the truncated system, this procedure leads to a matrix equation for the vector of c_0. We then have,

$$(\mathrm{M}_0 + \mathrm{M}_1)\mathrm{c}_0 = \mathrm{F}$$

where $\mathbf{M}_0$ is a diagonal matrix with the elements

$$D\left(k_m^{\,2} + \kappa_n^{\,2}\right)^2 - m_0\omega^2$$

and

$$\mathrm{M}_1 = \mathrm{J}^{\mathrm{t}} \begin{bmatrix} \ddots & & \\ & EIk_m{}^4 - m_1\omega^2 & \\ & & \ddots \end{bmatrix} \mathrm{J}$$

The matrix transpose is denoted by t. The coefficients can be computed simply inverting the matrix given in the above equation.

2.3 Structural intensity

The previous section shows the deflection of the joist-floor in terms of the summation over the modes of spatial wavenumbers (k_{m}, κ_n). The structural intensity is then expressed in terms of the angle, at which the intensity propagates in the structure. We show that the bending wave energy is angular dependent, therefore the

The structural intensity of a thin (Kirchhof) plate, which has zero in-plane deformation, is given by the time-average of the following (see section 3.9 in [8]),

$$\Pi_x = D\left[\left(w_{xx} + \nu w_{yy}\right)\dot{w}_x^* + (1-\nu)\, w_{xy}\,\dot{w}_y^* - \nabla^2 w_x\,\dot{w}^*\right]$$

$$\Pi_y = D\left[\left(w_{yy} + \nu w_{xx}\right)\dot{w}_y^* + (1-\nu)\, w_{xy}\,\dot{w}_x^* - \nabla^2 w_y\,\dot{w}^*\right]$$

where dot and * indicate the time derivative and the complex conjugate, respectively.

The wave numbers (k_{m}, κ_n) can be rewritten using the radial coordinates,

$$k_m = l_{mn}\cos\theta_{mn},\ \ \kappa_n = l_{mn}\sin\theta_{mn},$$

$$l_{mn} = \sqrt{k_m{}^2 + \kappa_n{}^2},\ \ \theta_{mn} = \arctan\frac{A}{B}\frac{n}{m}$$

We then use the mode expressed using the above radial representation of $\phi_m\psi_n$ to express the deflection in terms of angular modes, that is,

$$\Phi_{mn}^a = \frac{1}{\sqrt{AB}} c_{mn} e^{il_{mn}\left(x\cos\theta_{mn} + y\sin\theta_{mn}\right)}$$

Then, we have

$$\Pi_x\left(\Phi_{mn}^a\right) = \frac{2\omega D}{AB}\left|c_{mn}\right|^2 l_{mn}{}^3\cos\theta_{mn}$$

Similarly, the y component of the intensity is

$$\Pi_y\left(\Phi_{mn}^a\right) = \frac{2\omega D}{AB}\left|c_{mn}\right|^2 l_{mn}{}^3\sin\theta_{mn}$$

From the result given so far, it can be seen that the plane wave intensity for a mode (mn) is

$$\Pi_{mn}\left(\theta_{mn}\right) = \frac{2\omega D}{AB}\left|c_{mn}\right|^2 l_{mn}{}^3.$$

In a finite sized medium the angles of propagation (as well as the wavenumbers and resonance frequencies) will be discrete. However, in order to say something more general, consider instead the average intensity in an angel band $\Delta\theta$. Thus, define the average angle dependent plane wave intensity as

$$\Pi(\theta) = \frac{1}{\Delta\theta}\int_\theta^{\theta+\Delta\theta} \sum_{m,n=1}^{\infty} \Pi_{mn}\left(\theta_{mn}\right)\delta\left(\theta - \theta_{mn}\right)d\theta$$

or simply

$$\Pi(\theta) = \frac{1}{\Delta\theta} \sum_{\theta_{mn} \in (\theta, \theta+\Delta\theta)} \Pi_{mn}(\theta_{mn})$$

From the above equation it is possible to study the angle dependent plane wave intensity for different structures. Figures 2 and 3 show the angular dependency of the averaged intensity.

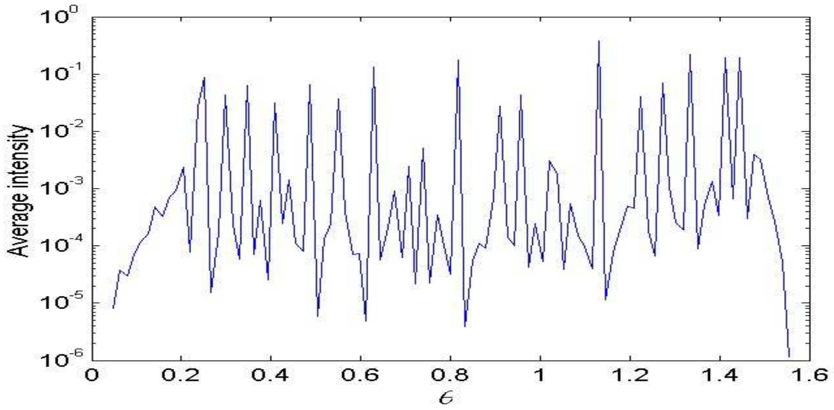

Figure 2: Average intensity in a rectangular plate, 7m by 3.2m, Young's modulus 14GPa, Poisson ratio 0.3, mass density per unit area 620 Kg

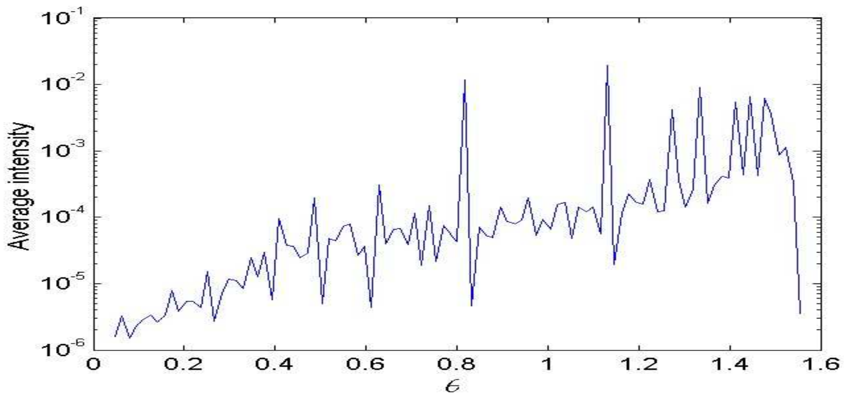

Figure 3: Average intensity of joist floor, same size as the plate in figure 2. Physical parameters of the joist are same as the plate.

3 SUMMARY

We have shown that the propagation of the energy in the joist-floor (ribbed structure) is directional dependent on the modal angles. Therefore the assumption of diffuseness cannot be justified for the structure of this kind. The numerical simulation of the vibration of the structure is simple. The result of the numerical simulation shows that the lightweight joist-floor structure needs to be modeled by taking each component into account rather than using the diffuseness assumption.

4 ACKNOWLEDGEMENTS

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